

$$Gx = \begin{pmatrix} -3 & 0 & 3 \\ -10 & 0 & 10 \\ -3 & 0 & 3 \end{pmatrix};$$

$$Gx1 = \begin{pmatrix} -3 \\ -10 \\ -3 \end{pmatrix};$$

$$Gx2 = (1 \ 0 \ -1);$$

$$img = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 2 & 2 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix};$$

The filter is separable.

Gx1.Gx2 // MatrixForm

$$\begin{pmatrix} -3 & 0 & 3 \\ -10 & 0 & 10 \\ -3 & 0 & 3 \end{pmatrix}$$

Response with the usual filter operation.

(* image is padded with zeros for border handling *)

responseDefault = ListConvolve[Gx, ArrayPad[img, 2]];

responseDefault // MatrixForm

$$\begin{pmatrix} 0 & -3 & 0 & 0 & 0 & 3 \\ -6 & -16 & 6 & 6 & 0 & 10 \\ -20 & -32 & 17 & 29 & 3 & 3 \\ -6 & -39 & -4 & 39 & 10 & 0 \\ 0 & -19 & -3 & 19 & 3 & 0 \\ 0 & -3 & 0 & 3 & 0 & 0 \end{pmatrix}$$

Filter convolve with Gx1 and store the result in a temporary variable.

tmp = ListConvolve[Gx1, ArrayPad[img, {{2, 2}, {0, 0}}]];

tmp // MatrixForm

$$\begin{pmatrix} 0 & -3 & 0 & -3 \\ -6 & -16 & 0 & -10 \\ -20 & -32 & -3 & -3 \\ -6 & -39 & -10 & 0 \\ 0 & -19 & -3 & 0 \\ 0 & -3 & 0 & 0 \end{pmatrix}$$

Apply Gx2 on the previous result.

responseSeparation = ListConvolve[Gx2, ArrayPad[tmp, {{0, 0}, {2, 2}}]];

responseSeparation // MatrixForm

$$\begin{pmatrix} 0 & -3 & 0 & 0 & 0 & 3 \\ -6 & -16 & 6 & 6 & 0 & 10 \\ -20 & -32 & 17 & 29 & 3 & 3 \\ -6 & -39 & -4 & 39 & 10 & 0 \\ 0 & -19 & -3 & 19 & 3 & 0 \\ 0 & -3 & 0 & 3 & 0 & 0 \end{pmatrix}$$

The responses are indeed identical.

responseDefault === responseSeparation

True